Definite Integration and Intro to Substitution

Math 130 - Essentials of Calculus

21 April 2021

Integral Terminology

In the integral

$$\int_a^b f(x)\,dx$$

- $f(x)$ is called the **integrand**
- a and b are called the **limits of integration**
- specifically, a is the **lower limit** and b is the **upper limit**

Total Cost

Suppose we are given the marginal cost function for a good: $M(q)$. How could we use the marginal cost to find the total cost?

Total Cost

Suppose we are given the marginal cost function for a good: $M(q)$. How could we use the marginal cost to find the total cost?
If we wanted to, for example, compute the cost of producing 100 of the product, we would compute

$$\int_0^{100} M(q)\, dq.$$

Total Cost

Suppose we are given the marginal cost function for a good: $M(q)$. How could we use the marginal cost to find the total cost?

If we wanted to, for example, compute the cost of producing 100 of the product, we would compute

$$\int_0^{100} M(q)\, dq.$$

This relationship works since the marginal cost function is just the derivative of the total cost function.

Total Cost

Suppose we are given the marginal cost function for a good: $M(q)$. How could we use the marginal cost to find the total cost?

If we wanted to, for example, compute the cost of producing 100 of the product, we would compute

$$\int_0^{100} M(q)\,dq.$$

This relationship works since the marginal cost function is just the derivative of the total cost function. In other words, total cost is an antiderivative of marginal cost!

Total Cost

Suppose we are given the marginal cost function for a good: $M(q)$. How could we use the marginal cost to find the total cost?

If we wanted to, for example, compute the cost of producing 100 of the product, we would compute

$$\int_0^{100} M(q)\,dq.$$

This relationship works since the marginal cost function is just the derivative of the total cost function. In other words, total cost is an antiderivative of marginal cost!

Using integration, we are even able to see how production costs would increase if we wanted to increase the amount of product produced. For example, if we wanted to see the additional costs involved in raising production from 200 to 400 units, we would just compute

$$\int_{200}^{400} M(q)\,dq.$$

Examples

Compute the following integrals

1. $\int_0^4 (6x - 5)dx$

Examples

Compute the following integrals

1. $\displaystyle\int_0^4 (6x - 5)dx$
2. $\displaystyle\int_{-1}^3 x^5 \, dx$

Examples

Compute the following integrals

1. $\int_0^4 (6x - 5)dx$
2. $\int_{-1}^3 x^5 \, dx$
3. $\int_3^5 (t^2 + 1)dx$

Examples

Compute the following integrals

1. $\displaystyle\int_0^4 (6x-5)dx$
2. $\displaystyle\int_{-1}^3 x^5\,dx$
3. $\displaystyle\int_3^5 (t^2+1)dx$
4. $\displaystyle\int_1^3 (12x-4x^3)dx$

Examples

Compute the following integrals

1. $\int_0^4 (6x - 5)dx$
2. $\int_{-1}^3 x^5 \, dx$
3. $\int_3^5 (t^2 + 1)dx$
4. $\int_1^3 (12x - 4x^3)dx$
5. $\int_1^9 4\sqrt{z} \, dz$

Examples

Compute the following integrals

1. $\displaystyle\int_0^4 (6x - 5)dx$
2. $\displaystyle\int_{-1}^3 x^5 \, dx$
3. $\displaystyle\int_3^5 (t^2 + 1)dx$
4. $\displaystyle\int_1^3 (12x - 4x^3)dx$
5. $\displaystyle\int_1^9 4\sqrt{z} \, dz$
6. $\displaystyle\int_{-1}^0 (2x - e^x)dx$

The Substitution Rule

Suppose the function f has an antiderivative F (that is, $F' = f$), then the chain rule says that

$$\frac{d}{dx}\left[F(g(x))\right] = F'(g(x))g'(x)$$

The Substitution Rule

Suppose the function f has an antiderivative F (that is, $F' = f$), then the chain rule says that

$$\frac{d}{dx}[F(g(x))] = F'(g(x))g'(x)$$

or, equivalently,

$$\frac{d}{dx}[F(g(x))] = f(g(x))g'(x).$$

The Substitution Rule

Suppose the function f has an antiderivative F (that is, $F' = f$), then the chain rule says that

$$\frac{d}{dx}\left[F(g(x))\right] = F'(g(x))g'(x)$$

or, equivalently,

$$\frac{d}{dx}\left[F(g(x))\right] = f(g(x))g'(x).$$

If we take the antiderivative of both sides, we end up with

$$\int f(g(x))g'(x)\,dx = F(g(x)) + C.$$

The Substitution Rule

Suppose the function f has an antiderivative F (that is, $F' = f$), then the chain rule says that

$$\frac{d}{dx}\left[F(g(x))\right] = F'(g(x))g'(x)$$

or, equivalently,

$$\frac{d}{dx}\left[F(g(x))\right] = f(g(x))g'(x).$$

If we take the antiderivative of both sides, we end up with

$$\int f(g(x))g'(x)\, dx = F(g(x)) + C.$$

Example

Compute the integral

$$\int 2xe^{x^2}\, dx$$

u-Substitution

The substitution rule is more commonly referred to as "*u*-substitution" because of the following way in which it is usually used:

u-Substitution

The substitution rule is more commonly referred to as "u-substitution" because of the following way in which it is usually used:

Theorem (u-Substitution)

If $u = g(x)$ is a differentiable function and f is continuous on the range of g, then

$$\int f(g(x))g'(x)\,dx = \int f(u)\,du.$$

u-Substitution

The substitution rule is more commonly referred to as "u-substitution" because of the following way in which it is usually used:

Theorem (u-Substitution)

If $u = g(x)$ is a differentiable function and f is continuous on the range of g, then

$$\int f(g(x))g'(x)\,dx = \int f(u)\,du.$$

If $u = g(x)$, then $\frac{du}{dx} = g(x)$ and so (by ignoring some details) we can say $du = g'(x)dx$.

u-Substitution

The substitution rule is more commonly referred to as "*u*-substitution" because of the following way in which it is usually used:

Theorem (*u*-Substitution)

If $u = g(x)$ is a differentiable function and f is continuous on the range of g, then

$$\int f(g(x))g'(x)\,dx = \int f(u)\,du.$$

If $u = g(x)$, then $\dfrac{du}{dx} = g(x)$ and so (by ignoring some details) we can say $du = g'(x)dx$. Then if $F' = f$, we have

$$\int f(\underbrace{g(x)}_{u})\,\underbrace{g'(x)\,dx}_{du} = \int f(u)\,du = F(u) + C.$$

Examples

1. $\displaystyle\int x^2\sqrt{x^3+1}\,dx$

Examples

1. $\int x^2\sqrt{x^3+1}\,dx$
2. $\int (3x-2)^{20}\,dx$

Examples

1. $\displaystyle\int x^2\sqrt{x^3+1}\,dx$
2. $\displaystyle\int (3x-2)^{20}\,dx$
3. $\displaystyle\int t(3-t^2)^4\,dt$
4. $\displaystyle\int \frac{x}{x^2+4}\,dx$

u-Substitution in Definite Integrals

Theorem (u-Substitution for Definite Integrals)

If g' is continuous on $[a, b]$ and f is continuous on the range of $u = g(x)$, then

$$\int_a^b f(g(x))g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du.$$

u-Substitution in Definite Integrals

Theorem (u-Substitution for Definite Integrals)

If g' is continuous on $[a, b]$ and f is continuous on the range of $u = g(x)$, then

$$\int_a^b f(g(x))g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du.$$

Example

Compute the integrals

1. $\displaystyle\int_0^1 \sqrt[3]{1+7x}\,dx$

u-Substitution in Definite Integrals

Theorem (u-Substitution for Definite Integrals)

If g' is continuous on $[a, b]$ and f is continuous on the range of $u = g(x)$, then

$$\int_a^b f(g(x))g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du.$$

Example

Compute the integrals

1. $\displaystyle\int_0^1 \sqrt[3]{1+7x}\,dx$
2. $\displaystyle\int_0^2 t^2\sqrt{8-t^3}\,dt$

u-Substitution in Definite Integrals

Theorem (u-Substitution for Definite Integrals)

If g' is continuous on $[a, b]$ and f is continuous on the range of $u = g(x)$, then

$$\int_a^b f(g(x))g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du.$$

Example

Compute the integrals

1. $\displaystyle\int_0^1 \sqrt[3]{1+7x}\,dx$
2. $\displaystyle\int_0^2 t^2\sqrt{8-t^3}\,dt$
3. $\displaystyle\int_1^3 4ze^{z^2-1}\,dz$
4. $\displaystyle\int_e^{e^4} \frac{dx}{x\ln x}\,dx$